

Statistics

Summer 2023

Lecture 17



Feb 19-8:47 AM

Testing One Population Standard Deviation:

$$H_0: \sigma = \sigma_0$$

$$H_0: \sigma \leq \sigma_0$$

$$H_0: \sigma \geq \sigma_0$$

$$H_1: \sigma \neq \sigma_0$$

$$H_1: \sigma > \sigma_0$$

$$H_1: \sigma < \sigma_0$$

TTT

RTT

LTT

Always identify the claim

use P-value method only

CTS Formula

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$$

use χ^2_{dS} with $dS = n-1$ to find P-value

Proceed with testing chart

Draw final conclusion about the claim

Jul 11-9:56 AM

Given: $H_0: \sigma = 20$, claim is H_1 , $\alpha = .02$

$n = 10, S = 15$

Test the claim.

$H_0: \sigma = 20$

$H_1: \sigma \neq 20$ claim, TTT

CTS

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(10-1) \cdot 15^2}{20^2}$$

$$= 5.063$$

$\chi^2_{cdf}(5.063, 9) = 0.829$

$\chi^2_{cdf}(0, 5.063, 9) = 0.171$

P-value = $2 * \text{Smaller area} = 2 * (.171) = 0.342$

P-value $>$ α
 $.342 > .02$

H_0 valid, H_1 invalid
 Invalid claim
 Reject the claim

Jul 12-7:33 AM

The college claims that standard deviation of salaries of all teachers is below \$750

$\sigma < 750$ claim
 H_1

In a sample of 12 teachers, standard deviation of their salaries was \$500.

$n = 12, S = 500$

Use this sample to test the claim. \rightarrow No α
 \Rightarrow use .05

$H_0: \sigma \geq 750$

$H_1: \sigma < 750$ LTT, claim

CTS

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$$

$$= \frac{(12-1) \cdot 500^2}{750^2}$$

$$= 4.889$$

P-value = $\chi^2_{cdf}(0, 4.889, 11) = 0.064$

P-value $>$ α
 $.064 > .05$

H_0 valid, H_1 invalid
 Invalid claim
 Reject the claim

If we choose α to be .1,
 then P-value $\leq \alpha$
 $.064 \leq .1$
 H_0 invalid \rightarrow Valid claim
 H_1 valid \rightarrow FTR the claim

Jul 12-7:42 AM

Given $H_0: \sigma \leq 5$, claim is H_1 , $\alpha = .1$
 $n = 10$, $S = 8$

Test the claim.

$H_0: \sigma \leq 5$

$H_1: \sigma > 5$ RTT, claim

CTS
 $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(10-1) \cdot 8^2}{5^2}$
 $= 23.04$

Area = P-value
 $= \chi^2_{cdf}(23.04, 9)$
 $= .006$

$df = n-1 = 9$

P-value $< \alpha$
 $.006 < .1$ H_0 invalid
 H_1 valid \rightarrow valid claim \rightarrow FTR the claim

If we choose $\alpha = .005$

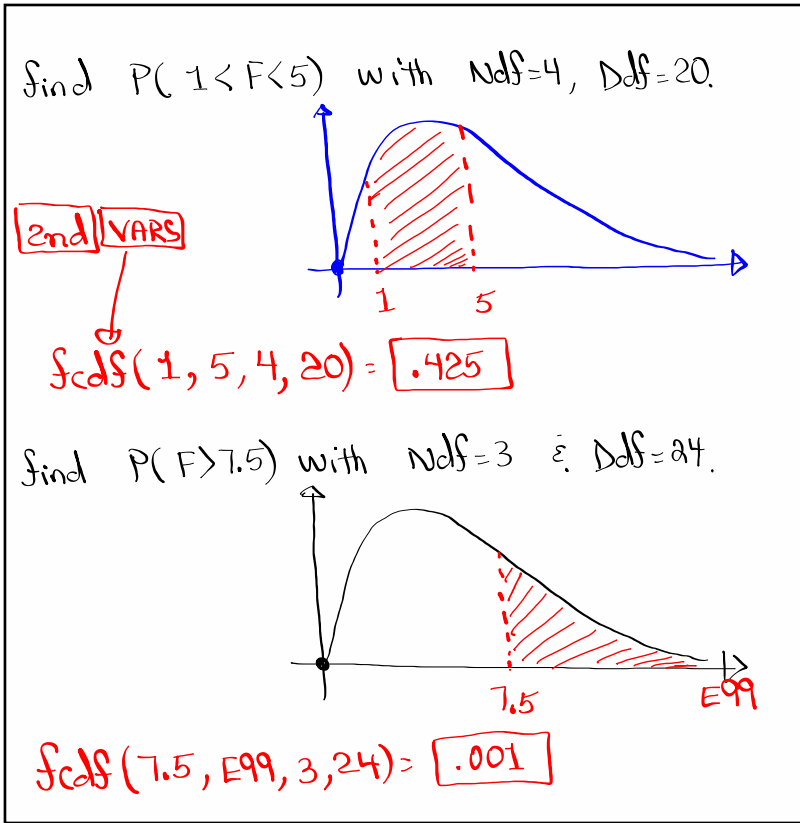
P-value $> \alpha$ $\Rightarrow H_0$ valid
 $.006 > .005$ H_1 invalid \rightarrow Invalid claim
 Reject the claim

Jul 12-7:53 AM

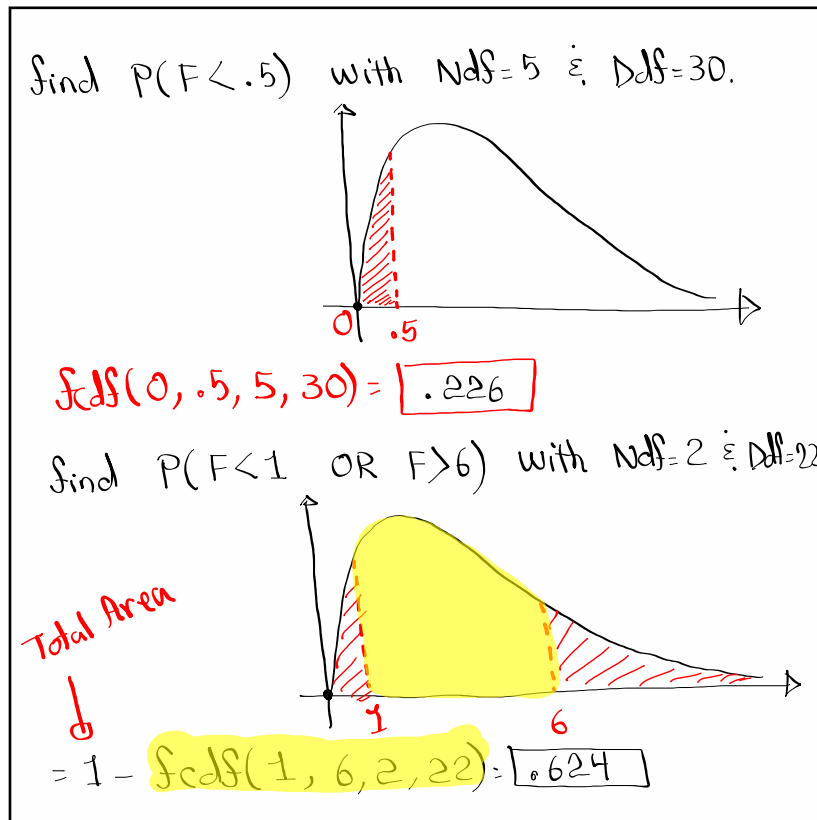
F-Dist:

- Graph is similar to χ^2 -Dist.
 Starts at 0.
 Skewed to the right
 Total area = 1
- It comes with two degrees of freedom
 Numerator df \rightarrow Ndf
 Denominator df \rightarrow Ddf
- We use Fedf to find prob.
 $Fcdf(L, U, Ndf, Ddf)$

Jul 12-8:31 AM

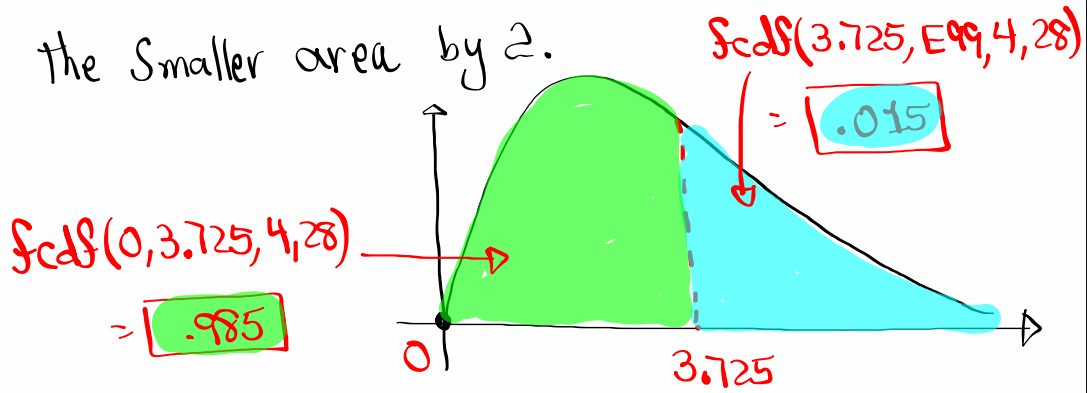


Jul 12-8:34 AM



Jul 12-8:39 AM

Find the area on each side of $F=3.725$ with $ndf=4$ & $Ddf=28$, then multiply the smaller area by 2.



$2 * \text{Smaller Area} = 2(.015) = .030$

Jul 12-8:44 AM

Comparing Two Population Standard Deviations:

$H_0: \sigma_1 = \sigma_2$

$H_0: \sigma_1 \leq \sigma_2$

$H_0: \sigma_1 \geq \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$

$H_1: \sigma_1 > \sigma_2$

$H_1: \sigma_1 < \sigma_2$

TTT

RTT

LTT

Always identify the claim

Group 1	Group 2
$n_1 =$	$n_2 =$
$S_1 =$	$S_2 =$

$S_1 > S_2$

$ndf = n_1 - 1$

$Ddf = n_2 - 1$

CTS $F = \frac{S_1^2}{S_2^2}$

use 2-Samp F Test to find CTS & P-value

using P-value method

Proceed with testing chart

Draw final conclusion about the claim

Jul 12-8:48 AM

Consider the chart below

Group 1	Group 2	1) Is $S_1 > S_2$? Yes 2) $Ndf = n_1 - 1 = 7$ $Ddf = n_2 - 1 = 9$
$n_1 = 8$	$n_2 = 10$	
$S_1 = 5$	$S_2 = 4$	

3) CTS $F = \frac{S_1^2}{S_2^2} = \frac{5^2}{4^2} = 1.563$

use $\alpha = .1$ to test the claim $\sigma_1 = \sigma_2$.

$H_0: \sigma_1 = \sigma_2$ claim
 $H_1: \sigma_1 \neq \sigma_2$ TTT

CTS $F = 1.563$
 P-value $P = .522$

P-value α
 $.522 > .1$

H_0 valid
 Valid claim
 FTR the claim

STAT TESTS
 2-Samp F Test

Inpt: Stats
 $S_1 = 5$
 $n_1 = 8$
 $S_2 = 4$
 $n_2 = 10$
 $\sigma_1 \neq \sigma_2$ H_1
 Calculate

Jul 12-8:55 AM

10 Female students had a stand. dev. of 8 for their ages. Females: $n = 10, S = 8$

12 Male students had a stand. dev. of 5 for their ages. Males: $n = 12, S = 5$

Females	Males	1) Verify $S_1 > S_2$ ✓ 2) $Ndf = n_1 - 1 = 9$ $Ddf = n_2 - 1 = 11$
$n_1 = 10$	$n_2 = 12$	
$S_1 = 8$	$S_2 = 5$	

3) CTS $F = \frac{S_1^2}{S_2^2} = \frac{8^2}{5^2} = 2.56$

Test the claim that $\sigma_1 \neq \sigma_2$.

$H_0: \sigma_1 = \sigma_2$ $\alpha = .05$
 $H_1: \sigma_1 \neq \sigma_2$ claim, TTT

P-value α
 $.144 > .05$

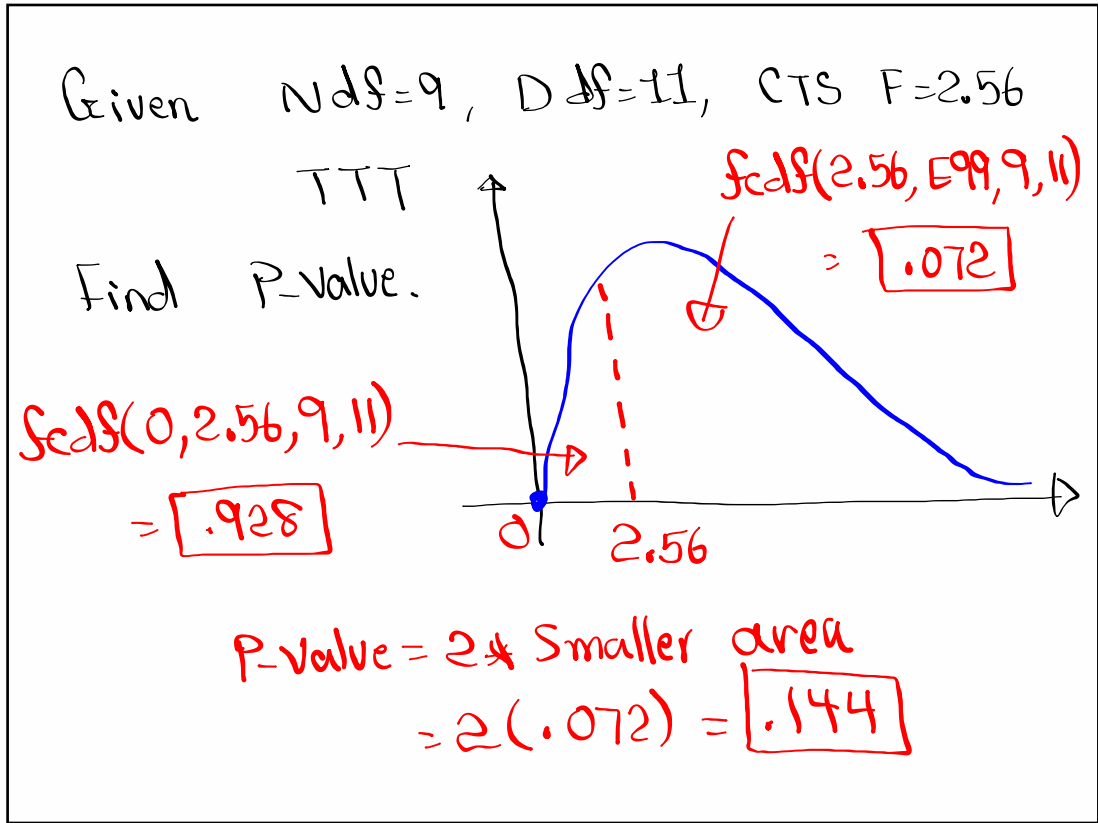
H_0 valid
 H_1 invalid
 Invalid claim
 Reject the claim

CTS $F = 2.56$
 P-value $P = .144$

2-Samp F Test

Inpt: Stats
 $S_1 = 8$
 $n_1 = 10$
 $S_2 = 5$
 $n_2 = 12$
 $\sigma_1 \neq \sigma_2$ H_1
 Calculate

Jul 12-9:03 AM



Jul 12-9:12 AM

Females: $n_1=5$, $S_1=10$

Males: $n_2=5$, $S_2=5$

$Ddf=4$

Females	Males
$n_1=5$	$n_2=5$
$S_1=10$	$S_2=5$

$Ndf=4 \rightarrow$

Test the claim $\sigma_F > \sigma_M$

$H_0: \sigma_1 \leq \sigma_2$ \rightarrow NO α \rightarrow use .05

$H_1: \sigma_1 > \sigma_2$ claim, RTT

$CTS F = 4$
 $P\text{-value } P = .104 \checkmark$

2-Samp F Test

$P\text{-value} > \alpha$
 $.104 > .05$

H_0 valid
 H_1 invalid

Invalid claim
Reject the claim

SG 31 ✓

Jul 12-9:16 AM

Consider the chart below for ages of randomly Selected nurses:

Females			Males		
32	40	28	36	40	35
35	45	25	45	48	
50	55				

$\bar{x} = 39$ $\bar{x} = 41$
 group 1 group 2
 $S = 11$ $S = 6$
 $n = 8$ $n = 5$

$H_0: \sigma_1 = \sigma_2$ claim
 $H_1: \sigma_1 \neq \sigma_2$ TTT
 P-value $\alpha = .05$
 .258
 H_0 valid, H_1 invalid
 valid claim \Rightarrow FTR the claim

Find \bar{x} , S , and n for each group.
 Round to whole #
 No $\alpha \rightarrow .05$
 Test the claim that there is no difference between Pop. Standard deviations.
 $\sigma_1 = \sigma_2$
 CTS $F = 3.361$
 P-value $P = .258$ ✓
 2-Samp F Test

\rightarrow Ndf = 7
 \rightarrow Ddf = 4

Jul 12-10:05 AM

Given CTS $F = 3.361$
 Ndf = 7
 Ddf = 4
 TTT
 Find P-value

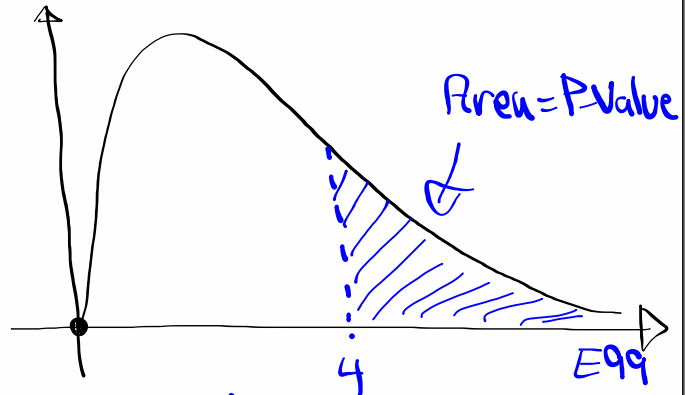
$Fcdf(3.361, E99, 7, 4) = .129$
 $Fcdf(0, 3.361, 7, 4) = .871$

P-value = 2 * Smaller area = 2(.129) = .258

Jul 12-10:15 AM

Given CTS $F=4$, RTT, $Ndf=4$, $Ddf=4$

Find P-value.



$$P\text{-value} = \text{fcdf}(4, E99, 4, 4) = \boxed{0.104}$$

Jul 12-9:22 AM